

MATHEMATICAL GAMES

On playing New Eleusis, the game that simulates the search for truth

by Martin Gardner

I shall always consider the best guesser the best Prophet.

Cicero, *De Divinatione*

Don't never prophesy—unless ye know.

James Russell Lowell,
The Biglow Papers

In June, 1959, I had the privilege of introducing in this department a remarkable simulation game called Eleusis. The game, which is played with an ordinary deck of cards, is named for the ancient Eleusinian mysteries, religious rites in which initiates learned a cult's secret rules. Hundreds of ingenious simulation games have been developed for modeling various aspects of life, but Eleusis is of special interest to mathematicians and scientists because it provides a model of induction, the process at the very heart of the scientific

method. My first column on Eleusis was reprinted in *The 2nd Scientific American Book of Mathematical Puzzles & Diversions* (Simon and Schuster, 1961). Since then Eleusis has evolved into a game so much more exciting to play than the original version that I feel I owe it to readers to bring them up to date. I will begin, however, with some history.

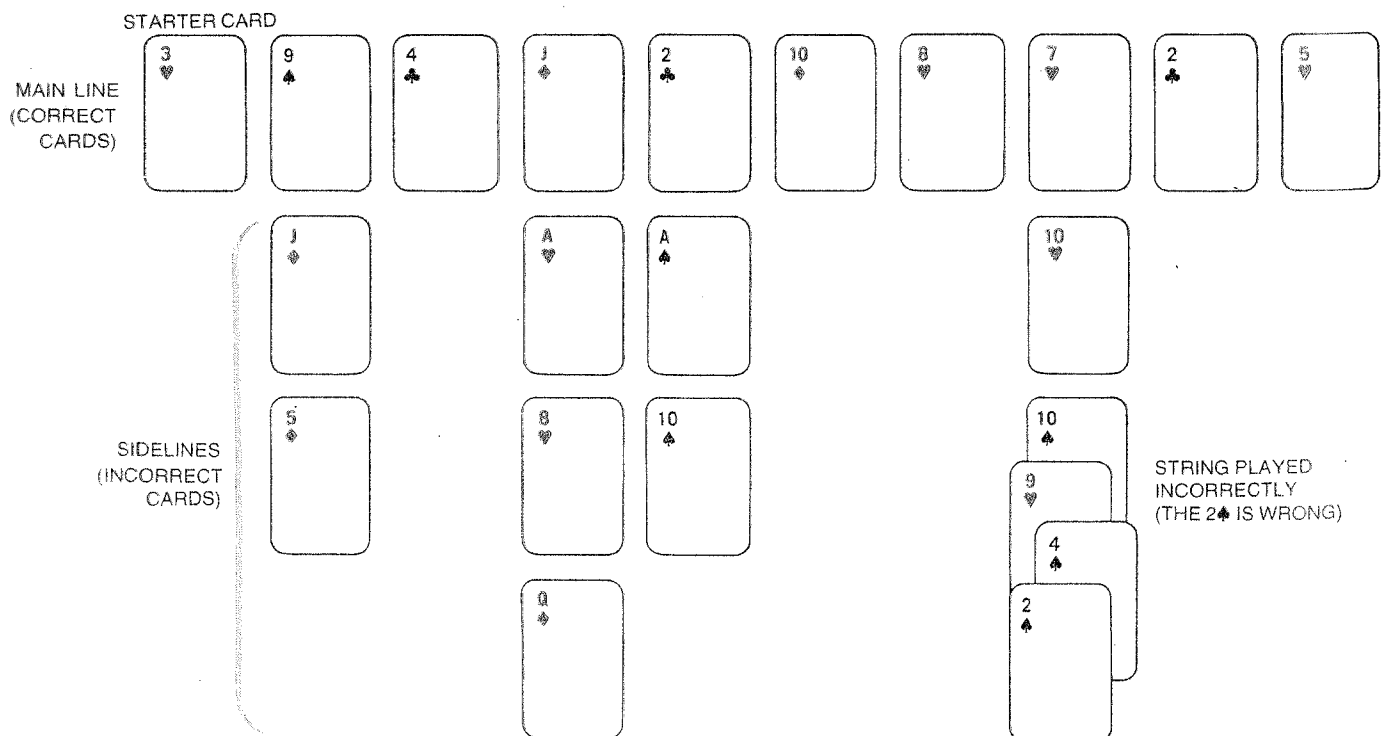
Eleusis was invented in 1956 by Robert Abbott of New York, who at the time was an undergraduate at the University of Colorado. He had been studying that sudden insight into the solution of a problem that psychologists sometimes call the "Aha" reaction. Great turning points in science often hinge on these mysterious intuitive leaps. Eleusis turned out to be a fascinating simulation of this facet of science, even though Abbott did not invent it with this in mind. In 1963 Abbott's complete rules for the game appeared in his book, *Abbott's New*

Card Games (hardcover, Stein & Day; paperback, Funk & Wagnalls).

Martin D. Kruskal, a distinguished mathematical physicist at Princeton University, became interested in the game and made several important improvements. In 1962 he published his rules in a monograph titled *Delphi: A Game of Inductive Reasoning*. Many college professors around the country used Eleusis to explain scientific method to students and to model the Aha process. Artificial intelligence scientists wrote computer programs for the game. At the System Development Corporation in Santa Monica research was done on Eleusis under the direction of J. Robert Newman. Litton Industries based a full-page advertisement on Eleusis. Descriptions of the game appeared in European books and periodicals. Abbott began receiving letters from all over the world with suggestions on how to make Eleusis a more playable game.

In 1973 Abbott discussed the game with John Jaworski, a young British mathematician who had been working on a computer version of Eleusis for teaching induction. Then Abbott embarked on a three-year program to reshape Eleusis, incorporating all the good suggestions he could. The new game is not only more exciting, its metaphorical level has been broadened as well. With the introduction of the roles of Prophet and False Prophet the game now simulates the search for any kind of truth. Here, then, based on a communication from Abbott, are the rules of New Eleusis as it is now played by aficionados.

At least four players are required. As many as eight can play but beyond that



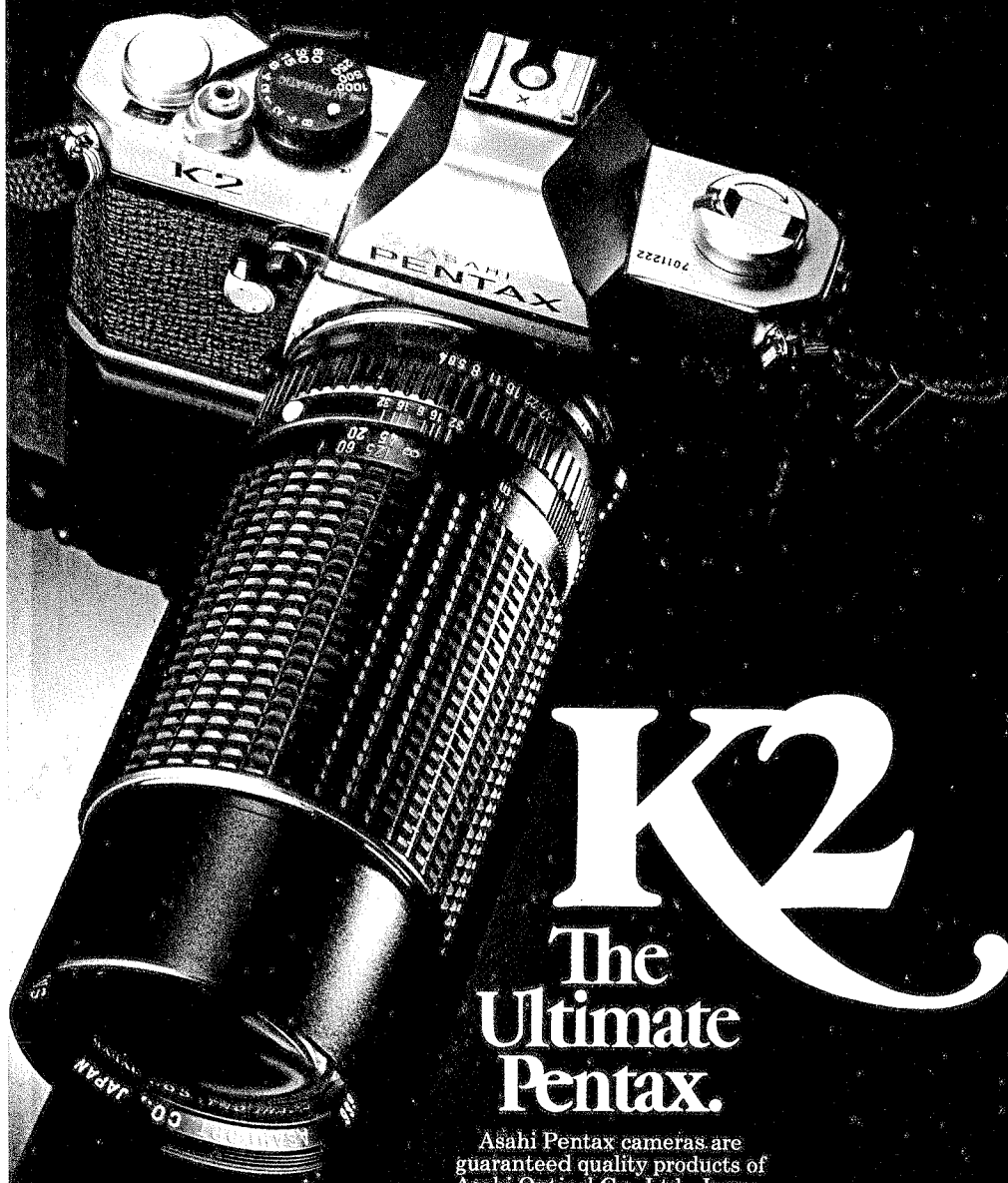
A typical round of Eleusis at an early stage

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the game becomes too long and chaotic.

Two standard decks, shuffled together, are used. (Occasionally a round will continue long enough to require a third deck.) A full game consists of one or more rounds (hands of play) with a different player dealing each round. The dealer may be called by such titles as God, Nature, Tao, Brahma, the Oracle (as in Delphi) or just Dealer.

The dealer's first task is to make up a "secret rule." This is simply a rule that defines what cards can be legally played during a player's turn. In order to do well players must figure out what the rule is. The faster a player discovers the rule, the higher his score will be.

One of the cleverest features of Eleusis is the scoring (described below) which makes it advantageous to the dealer to invent a rule that is neither too easy to guess nor too hard. Without this feature dealers would be tempted to formulate such complex rules that no one would guess them, and the game would become dull and frustrating.

An example of a rule that is too simple is: "Play a card of a color different from the color of the last card played." The alternation of colors would be immediately obvious. A better rule is: "Play so that primes and nonprimes alternate." For mathematicians, however, this might be too simple. For anyone else it might be too difficult. An example of a rule that is too complicated is: "Multiply the values of the last three cards played and divide by 4. If the remainder is 0, play a red card or a card with a value higher than 6. If the remainder is 1, play a black card or a picture card. If the remainder is 2, play an even card or a card with a value lower than 6. If the remainder is 3, play an odd card or a 10." No one will guess such a rule, and the dealer's score will be low.

Here are three examples of good rules for games with inexperienced players:

1. If the last legally played card was odd, play a black card. Otherwise play a red one.

2. If the last legally played card was black, play a card of equal or higher value. If the last card played was red, play a card of equal or lower value. (The values of the jack, queen, king and ace are respectively 11, 12, 13 and 1.)

3. The card played must be either of the same suit or the same value as the last card legally played.

The secret rules must deal only with the sequence of legally played cards. Of course, advanced players may use rules that refer to the entire pattern of legal and illegal cards on the table, but such rules are much harder to guess and are not allowed in standard play. Under no circumstances should the secret rule depend on circumstances external to the cards. Examples of such improper rules are those that depend on the sex of the last player, the time of day, whether God scratches his (or her) ear and so on.

The secret rule must be written down in unambiguous language, on a sheet of paper that is put aside for future confirmation. As Kruskal proposed, the dealer may give a truthful hint before the play begins. For example, he may say "Suits are irrelevant to the rule," or "The rule depends on the two previously played cards."

After the secret rule has been recorded, the dealer shuffles the double deck and deals 14 cards to each player and none to himself. He places a single card called the "starter" at the extreme left of the playing surface, as is indicated in the illustration on page 18. To determine who plays first the dealer counts clockwise around the circle of players, starting with the player on his left and excluding himself. He counts until he reaches the number on the starter card. The player indicated at that number begins the play that then continues clockwise around the circle.

A play consists of placing one or more cards on the table. To play a single card the player takes a card from his hand and shows it to everyone. If according to the rule the card is playable, the dealer says "Right." The card is then placed to the right of the starter card, on the "main line" of correctly played cards extending horizontally to the right.

If the card fails to meet the rule, the dealer says "Wrong." In this case the card is placed directly below the last card played. Vertical columns of incorrect cards are called "sidelines." (Kruskal introduced both the layout and the terminology of the main line and sidelines.) Thus consecutive incorrect plays extend the same sideline downward. If a player displays a wrong card, the dealer gives him two more cards as a penalty, thereby increasing his hand.

If a player thinks he has discovered the secret rule, he may play a "string" of 2, 3 or 4 cards at once. To play a string he overlaps the cards slightly to preserve their order and shows them to everyone. If all the cards in the string conform to the rule, the dealer says "Right." Then all the cards are placed on the main line with no overlapping, as if they were correctly played single cards.

If one or more cards in a string are wrong, the dealer declares the entire string wrong. He does not indicate which cards do not conform to the rule. The wrong cards are left overlapping to keep their identity as a string and the entire string, goes below the last card played. The player is then dealt twice as many cards as there are in the string.

The layout shown in the illustration on page 18 demonstrates all the rules of Eleusis mentioned so far. The dealer's secret rule for this layout is the first of the three given above.

Players improve their score by getting rid of as many cards as possible, and of course they can do this best if they guess the secret rule. At the start of a round

there is little information to go on, and plays are necessarily random. As the round continues and more and more information is added to the layout the rule becomes steadily easier to guess.

It may happen that a player thinks he knows the secret rule but finds he has no card that can be legally played. He then has the option of declaring "No play." In this case he shows his hand to everyone. If the dealer declares him right and his hand contains four cards or less, the cards are returned to the deck and the round ends. If he is right and has five or more cards, then his cards are put back into the deck, and he is dealt a fresh hand with four fewer cards than he previously held.

If the player is wrong in declaring no play, the dealer takes one of his correct cards and puts it on the main line. The player keeps the rest of his hand and, as a penalty, is dealt five more cards. A player who thinks he has no correct play but has not figured out the secret rule should realize that the odds are against his using the no play option successfully. He would do better to play a card at random.

When a player thinks he knows the secret rule, he has the opportunity to prove it and increase his score. He does so by declaring himself a Prophet. The Prophet immediately takes over the dealer's duties, calling plays right or wrong and dealing penalty cards when the others play. He can declare himself a Prophet only if all the following conditions prevail:

1. He has just played (correctly or incorrectly), and the next player has not played.

2. There is not already a Prophet.

3. At least two other players beside himself and the dealer are still in the round.

4. He has not been a Prophet before in this round.

When a player declares himself a Prophet, he puts a marker on the last card he played. A chess king or queen may be used. The Prophet keeps his hand but plays no more cards unless he is overthrown. The play continues to pass clockwise around the players' circle, skipping the Prophet.

Each time a player plays a card or string the Prophet calls the play right or wrong. The dealer then either validates or invalidates the Prophet's statement by saying "Correct" or "Incorrect." If the Prophet is correct, the card or string is placed on the layout—on the main line if right, on a sideline if wrong—and the Prophet gives the player whatever penalty cards are required.

If the dealer says "Incorrect," the Prophet is instantly overthrown. He is declared a False Prophet. The dealer removes the False Prophet's marker and gives him five cards to add to his hand. He is not allowed to become a Prophet again during the same round, although

any other player may do so. The religious symbolism is obvious, but as Abbott points out there is also an amusing analogy here with science: "The Prophet is the scientist who publishes. The False Prophet is the scientist who publishes too early." It is the fun of becoming a Prophet and of overthrowing a False Prophet that is the most exciting feature of New Eleusis.

After a Prophet's downfall the dealer takes over his former duties. He completes the play that overthrew the Prophet, placing the card or string in its proper place on the layout. If the play is wrong, however, no penalty cards are given. The purpose of this exemption is to encourage players to make unusual plays—even deliberately wrong ones—in the hope of overthrowing the Prophet. In Karl Popper's language, it encourages scientists to think of ways of "falsifying" a colleague's doubtful theory.

If there is a Prophet and a player believes he has no card to play, things get a bit complicated. This seldom happens, and so you can skip this part of the rules now and refer to it only when the need arises. There are four possibilities once the player declares no play:

1. Prophet says "Right," dealer says "Correct." The Prophet simply follows the procedure described earlier.

2. Prophet says "Right," dealer says "Incorrect." The Prophet is immediately overthrown. The dealer takes over and handles everything as usual, except that the player is not given any penalty cards.

3. Prophet says "Wrong," dealer says "Incorrect." In other words, the player is right. The Prophet is overthrown, and the dealer handles the play as usual.

4. Prophet says "Wrong," dealer says "Correct." In this case the Prophet now must pick one correct card from the player's hand and put it on the main line. If he does this correctly, he deals the player the five penalty cards and the game goes on. It is possible, however, for the Prophet to make a mistake at this point and pick an incorrect card. If that happens, the Prophet is overthrown. The wrong card goes back into the player's hand and the dealer takes over with the usual procedure, except that the player is not given penalty cards.

After 30 cards have been played, and there is no Prophet in the game, players are expelled from the round when they make a wrong play, that is, if they play a wrong card or make a wrong declaration of no play. An expelled player is given the usual penalty cards for his final play and then drops out of the round, retaining his hand for scoring.

If there is a Prophet, expulsions are delayed until at least 20 cards have been laid down after the Prophet's marker. Chess pawns are used as markers so that it is obvious when expulsion is possible. As long as there is no Prophet a white pawn goes on every 10th card placed on

the layout. If there is a Prophet, a black pawn goes on every 10th card laid down after the Prophet's marker. When a Prophet is overthrown, the black pawns and the Prophet's marker are removed.

A round can therefore go in and out of the phase when expulsions are possible. For example, if there are 35 cards on the layout and no Prophet, Smith is expelled when he plays incorrectly. Next Jones plays correctly and declares herself a Prophet. If Brown then plays incorrectly, she is not expelled because 20 cards have not yet been laid down after the Prophet's marker.

A round can end in two ways: (1) when a player runs out of cards or (2) when all players (excluding a Prophet, if there is one) have been expelled.

The scoring in Eleusis is as follows:

1. The greatest number of cards held by anyone (including the Prophet) is called the "high count." Each player (including the Prophet) subtracts the number of cards in his hand from the high count. The difference is his score. If he has no cards, he gets a bonus of four points.

2. The Prophet, if there is one, also gets a bonus. It is the number of main-line cards that follow his marker plus twice the number of sideline cards that follow his marker, that is, a point for each correct card since he became a Prophet and two points for each wrong card.

3. The dealer's score equals the high score of any player. There is one exception: If there is a Prophet, count the number of cards (right and wrong) that precede the Prophet's marker and double this number; if the result is smaller than the highest score, the dealer's score is that smaller number.

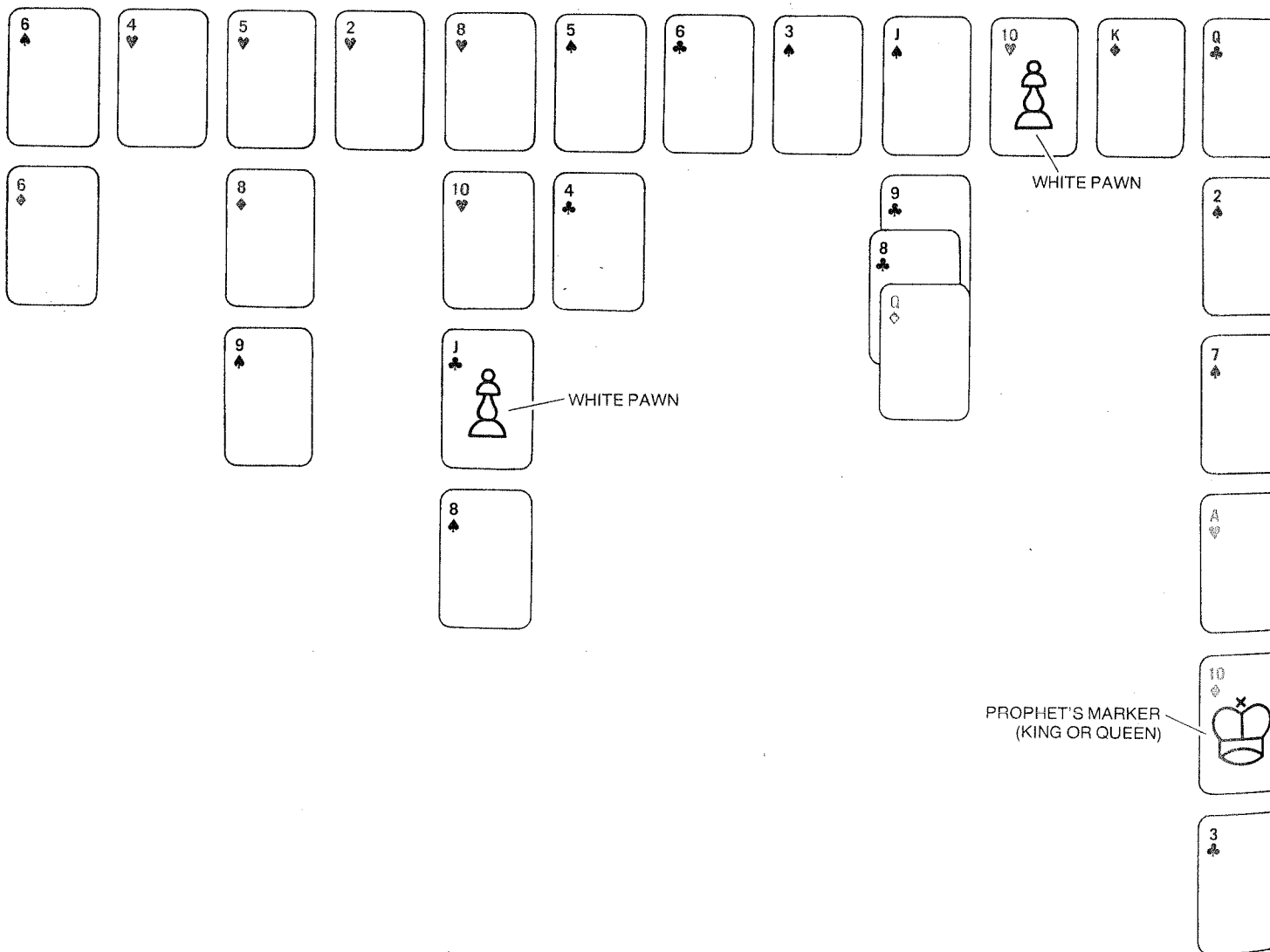
If there is time for another round, a new dealer is chosen. In principle the game ends after every player has been dealer, but this could take most of a day. To end the game before everyone has dealt, each player adds his scores for all the rounds played plus 10 more points if he has not been a dealer. This compensates for the fact that dealers tend to have higher than average scores.

The layout in the illustration on these two pages shows the end of a round

with five players. Smith was the dealer. The round ended when Jones got rid of her cards. Brown was the Prophet and ended with nine cards. Robinson was expelled when he incorrectly played the 10 of spades; he had 14 cards. Adams had 17 cards at the end of the game.

The high count is 17. Therefore Adams' score is 17 minus 17, or zero. Robinson's score is 17 minus 14, or three. Jones receives 17 minus zero, or 17, plus a four-point bonus for having no cards, so that her score is 21. Brown gets 17 minus nine, or eight, plus the Prophet's bonus of 34 (12 main-line and 11 sideline cards following his marker), making a total score of 42. This is the highest score for the round. Twice the number of cards preceding the Prophet's marker is 50. Smith, the dealer, receives 42 because it is the smaller of the two numbers 42 and 50.

Readers are invited to look over this layout and see if they can guess the secret rule. The play has been standard, so that the rule is confined strictly to the main-line sequence. I shall give the secret rule next month.



Layout at the end of a round of Eleusis includes a main line, several sidelines and various markers. White chess pawns

Some miscellaneous advice from Abbott should help inexperienced Eleusis players. Since layouts tend to be large, the best way to play the game is on the floor. Of course a large table can be used as well as miniature cards on a smaller table. If necessary, the main line can be broken on the right and continued below on the left.

Remember that in Eleusis the dealer maximizes his score by choosing a rule that is neither too easy nor too difficult. Naturally this depends both on how shrewdly the dealer estimates the ability of the players and how accurately he evaluates the complexity of his rule. Both estimates require considerable experience. Beginning players tend to underestimate the complexity of their rules.

For example, the rule used in the first layout is simple. Compare it with: "Play a red card, then a black card, then an odd card, then an even card and repeat cyclically." This rule seems to be simpler, but in practice the shift from the red-black variable to the even-odd variable makes it difficult to discover. Abbott points out that in general restrictive

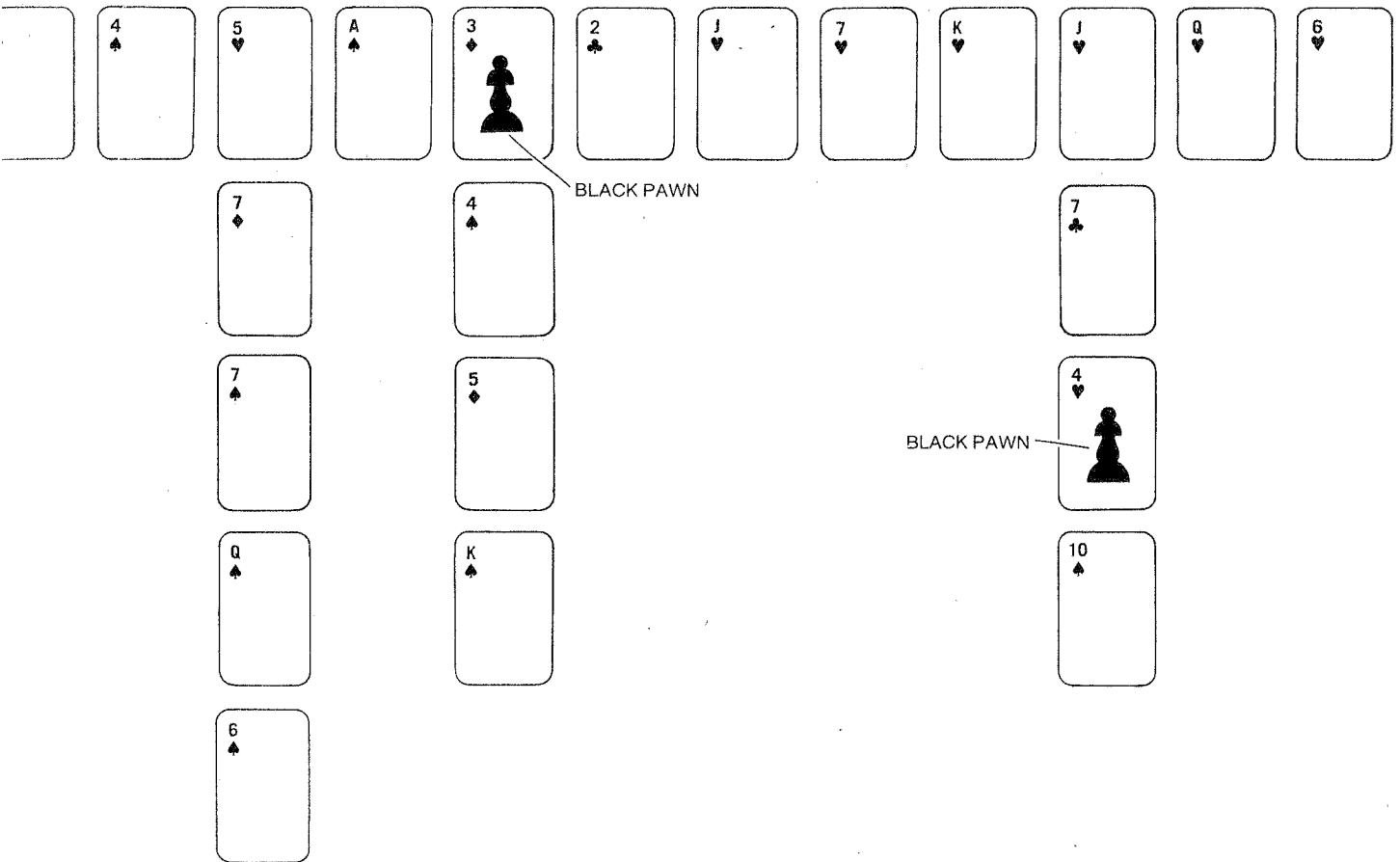
rules that allow only about a fourth of the cards to be acceptable on any given play are easier to guess than less restrictive rules that allow half or more of the cards to be acceptable.

I shall not belabor the ways in which the game models a search for truth (scientific, mathematical or metaphysical) since I discussed them in my first column on the game. I shall add only the fantasy that God or Nature may be playing thousands, perhaps a countless number, of simultaneous Eleusis games with intelligences on planets in the universe, always maximizing his or her pleasure by a choice of rules that the lesser minds will find not too easy and not too hard to discover if given enough time. The supply of cards is infinite, and when a player is expelled, there are always others to take his place.

Prophets and False Prophets come and go, and who knows when one round will end and another begin? Searching for any kind of truth is an exhilarating game. It is worth remembering that there would be no game at all unless the rules were hidden.

Readers can send \$1 to Robert Abbott and receive postpaid a more detailed set of instructions for playing Eleusis; they will also be placed on his mailing list and receive any future developments of the game. Abbott's address is Box 1175, General Post Office, New York, N.Y. 10001.

Last month I asked whether or not the straight-line shadow on the photograph of the McDonnell Planetarium in St. Louis corresponds to one of the straight lines that rule the surface of the structure's hyperboloid of revolution of one sheet. The answer is yes. At those times when the line containing the edge of the shadow intersects the sun, the edge of the shadow on the side of the planetarium coincides with a generating line of the surface. That line is a projection of a single point on the circumference of the circular top of the planetarium, not a projection of the entire circumference. When the sun is at any other elevation, the edge of the shadow will be a curved line and not a generating line of the surface.



placed on every 10th card played in the round and black pawns on every 10th card played after a Prophet's marker